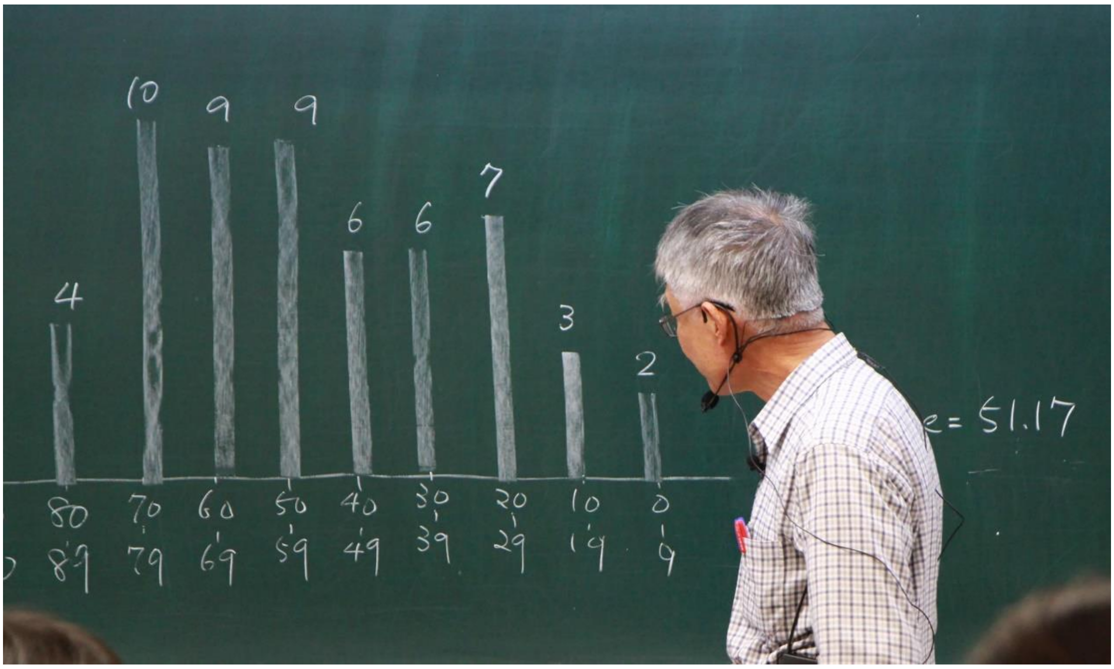
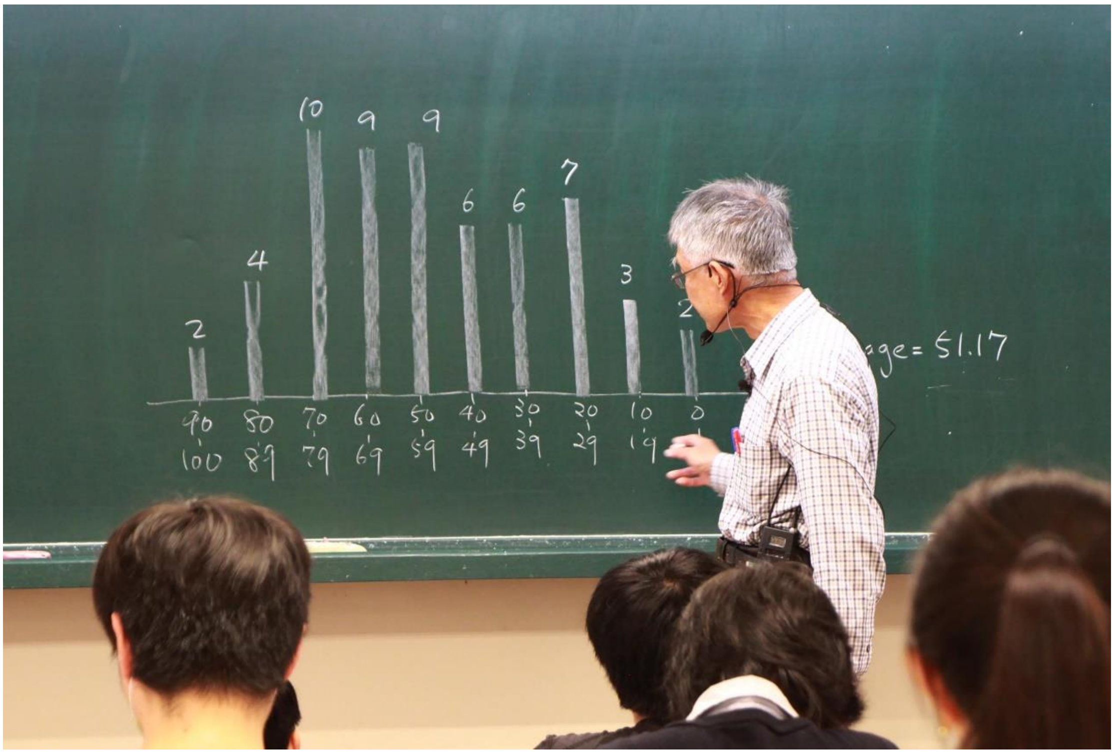
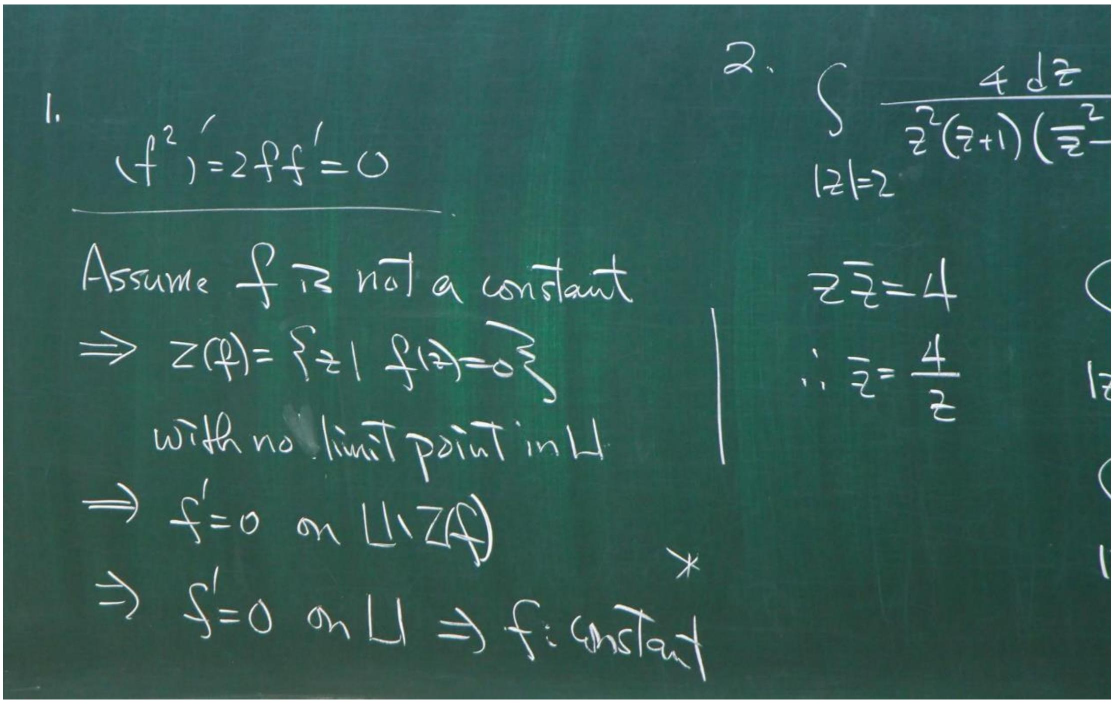
【10920 程守慶教授複變數函數論 / 第 10 堂版書】





2 4 10 9 9 6 6 7 3 2
90 100
80 89
70 79
60 69
50 59
40 49
30 39
20 29
10 19
age= 51.17



1.
(f²)′=2ff′=0
Assume f is not a constant
⇒ Z(f)= {z | f(z)=0}
with no limit point in U
⇒ f′=0 on U\Z(f)
⇒ f′=0 on U ⇒ f: constant
2.
|z|=2
zz̄=4
∴ z̄ = 4/z

2. $$\oint_{|z|=2} \frac{4\,dz}{\bar{z}^2(z+1)(\bar{z}^2-5\bar{z}+4)}$$

$\bar{z}^2-5\bar{z}+4=(\bar{z}-4)(\bar{z}-1)$

Res at $1 = -\frac{1}{6}$

Res at $-1 = \frac{1}{10}$

$z\bar{z}=4$

$\therefore \bar{z}=\frac{4}{z}$

$$= \oint_{|z|=2} \frac{4\,dz}{\bar{z}^2(z+1)\left(\frac{16}{z^2}-\frac{20}{z}+4\right)}$$

$$= \oint_{|z|=2} \frac{4\,dz}{(z+1)(4z^2-20z+16)}$$

$$= \oint_{|z|=2} \frac{dz}{(z+1)(z^2-5z+4)}$$

$$= \oint_{|z|=2} \frac{dz}{(z+1)(z-1)(z-4)} = 2\pi i\left(\frac{1}{10}-\frac{1}{6}\right)$$

$$= 2\pi i\,\frac{3-5}{30}$$

$$= -\frac{2\pi i}{15}$$

(1, −1) 4

at $1 = -\frac{1}{6}$

at $-1 = \frac{1}{10}$

$i\left(\frac{1}{10}-\frac{1}{6}\right)$

$i\,\frac{3-5}{30}$

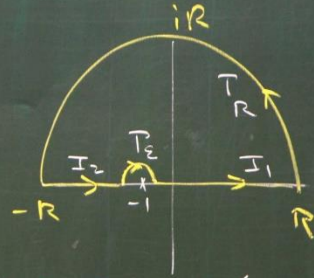
$\frac{2\pi i}{15}$

3. $$\text{P.V.}\int_{-\infty}^{\infty} \frac{dx}{x^3+4x+5}$$

$x^3+4x+5=(x+1)(x^2-x+5)$

$x=-1$

$x=\frac{1\pm\sqrt{19}i}{2}$

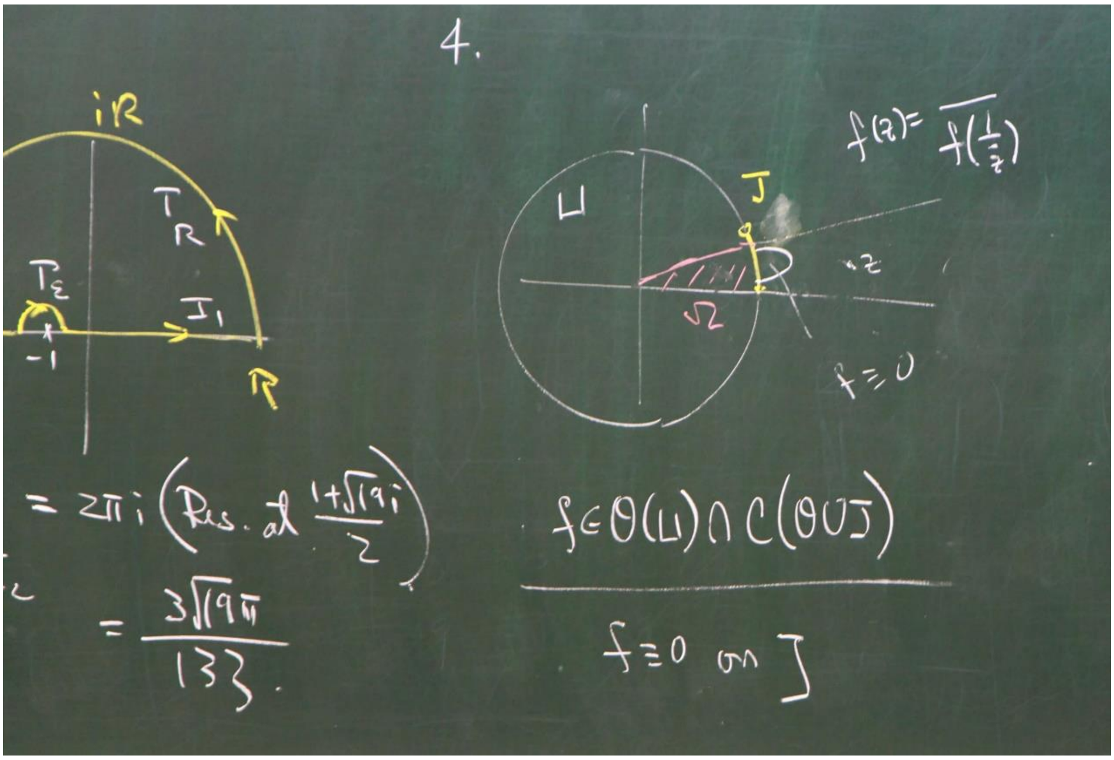


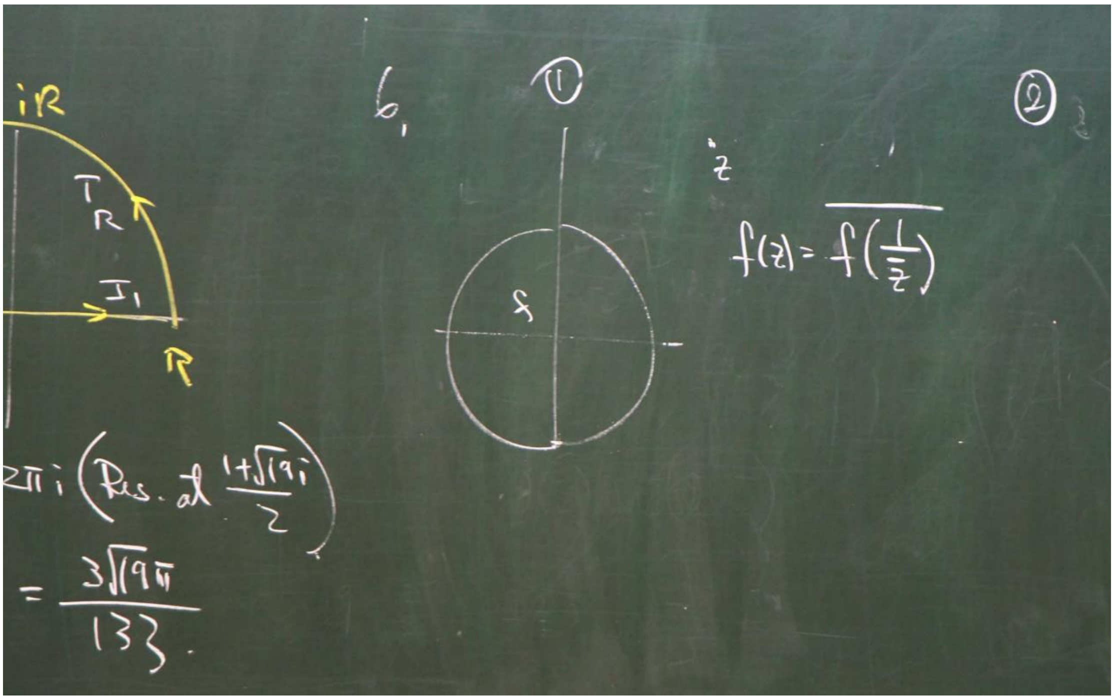
$$\int_{\Gamma_R}+\int_{\Gamma_\varepsilon}+\int_{I_1}+\int_{I_2} = 2\pi i\left(\text{Res. at } \frac{1+\sqrt{19}i}{2}\right)$$

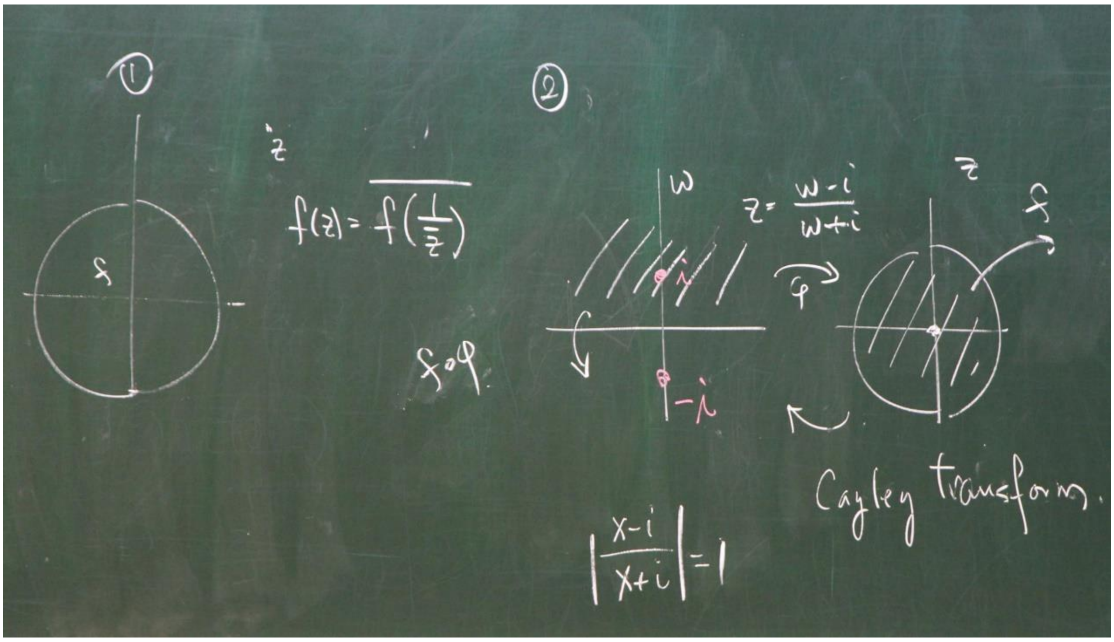
$$= \frac{3\sqrt{19}\pi}{133}.$$

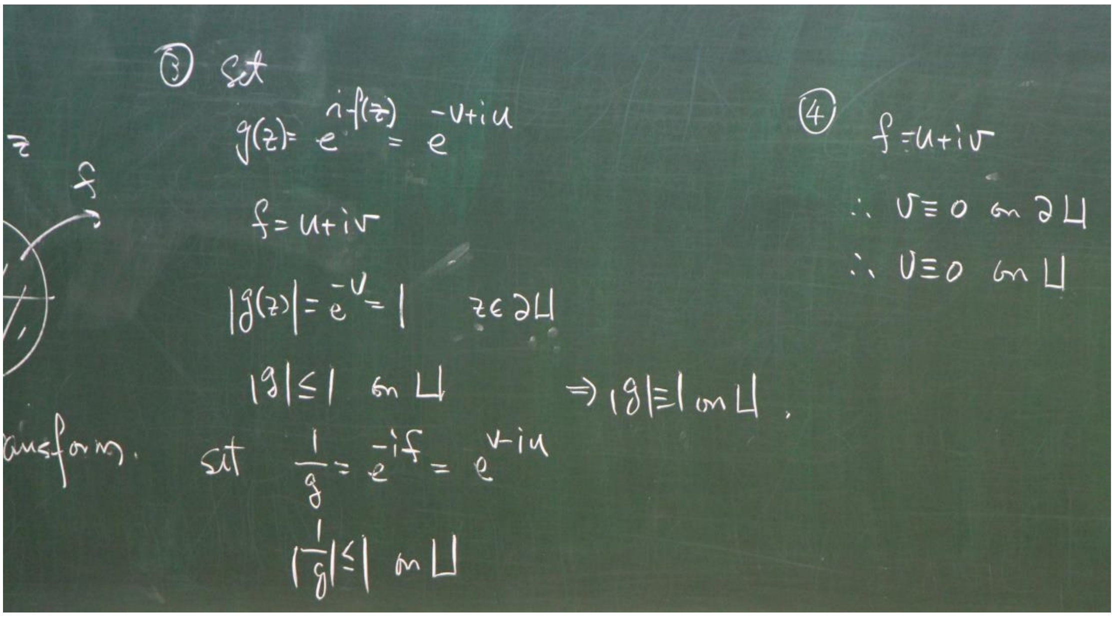
$\int_{\Gamma_R} \to 0$

$\int_{\Gamma_\varepsilon} \to -\frac{\pi}{7}i$


4.
iR
T_R
T_ε
J_1
-1
R
$= 2\pi i \left(\text{Res. at } \frac{1+\sqrt{19}i}{2}\right)$
$= \frac{3\sqrt{19}\pi}{133}.$
$f(z) = \overline{f(\frac{1}{\bar z})}$
U
J
Ω
z
$f \equiv 0$
$f \in \mathcal{O}(U) \cap C(U \cup J)$
$f \equiv 0$ on J


6.
①
②
iR
T_R
J_1
R
$2\pi i \left(\text{Res. at } \frac{1+\sqrt{19}i}{2}\right)$
$= \frac{3\sqrt{19}\pi}{133}.$
z
$f(z) = \overline{f(\frac{1}{\bar z})}$
f


①
f
z
$f(z) = \overline{f\left(\frac{1}{\bar{z}}\right)}$
$f \circ \varphi$
②
w
i
−i
$z = \frac{w-i}{w+i}$
φ
z
f
Cayley transform.
$\left|\frac{x-i}{x+i}\right| = 1$


③ set
$g(z) = e^{if(z)} = e^{-v+iu}$
$f = u + iv$
$|g(z)| = e^{-v} = 1 \quad z \in \partial \mathbb{H}$
$|g| \leq 1$ on $\mathbb{H}$ $\Rightarrow |g| \equiv 1$ on $\mathbb{H}$.
set $\frac{1}{g} = e^{-if} = e^{v-iu}$
$\left|\frac{1}{g}\right| \leq 1$ on $\mathbb{H}$
④
$f = u + iv$
$\therefore v \equiv 0$ on $\partial \mathbb{H}$
$\therefore v \equiv 0$ on $\mathbb{H}$
z
f
ansform.

Thm (H. Cartan).

Ω : bounded domain

Henri $\quad z_0 \in \Omega$

$f : \Omega \to \Omega$ holo

1904 – 2008 $\quad f(z_0) = z_0 \quad \Longrightarrow f(z) \equiv z$

$f'(z_0) = 1$

$z_0 = 0$

1980. Wolf

main

olo $\Longrightarrow f(z) \equiv z$

$z_0 = 0$

$$f(z) = z + \overset{0}{\overset{\shortparallel}{a_{k_0}}} z^{k_0} + \cdots$$

$$f \circ f(z) = f(z) + a_{k_0} (f(z))^{k_0} + \cdots$$

$$= z + 2 a_{k_0} z^{k_0} + \cdots$$

$$f^{\circ(m)}(z) = z + m a_{k_0} z^{k_0} + \cdots \quad : \Omega \to \Omega$$

$$|m a_{k_0}| \leq \frac{M}{r^{k_0}}$$

Schwarz lemma

$f: U \to U$. $f(0)=0$

Then ① $|f(z)| \le |z|$. $\forall z \in U$.

② $|f'(0)| \le 1$.

in addition. if "equality" holds in ① for some $z \in U$
or in ②

Then $f(z) = \lambda z$. for some $|\lambda| = 1$. $\lambda = e^{i\theta}$

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pf. Set

$$g(z) = \frac{f(z)}{z} \in \mathcal{O}(U)$$

$$\lim_{z\to 0} g(z) = \lim_{z \to 0} \frac{f(z)-f(0)}{z-0} = f'(0)$$

For $0 < r < 1$

$$|g(z)| \le \frac{1}{r} \qquad |z| \le r$$

$r \to 1^-$

$\therefore |g(z)| \le 1$, $z \in U \Rightarrow |f(z)| \le |z|$

$f: U \to U$

$f(0) = 0$

$f'(0) = 1$

$\Downarrow$

$f(z) = z$